WESLEY COLLEGE
By daring & by doing

YEAR 12 MATHEMATICS METHODS

Differentiation, applications and anti-differentiation

Test 2

Name: _____ Marks: /50

Calculator Free (24 marks)

1. [3 marks]

Using calculus techniques, find two numbers whose difference is 32 and whose product is a minimum.

[3]

2. [4 marks]

The displacement for an object is given by $x = \dfrac{2t-5}{3t+1}$, where x is in metres and t is in seconds. Find the equations for velocity and acceleration.

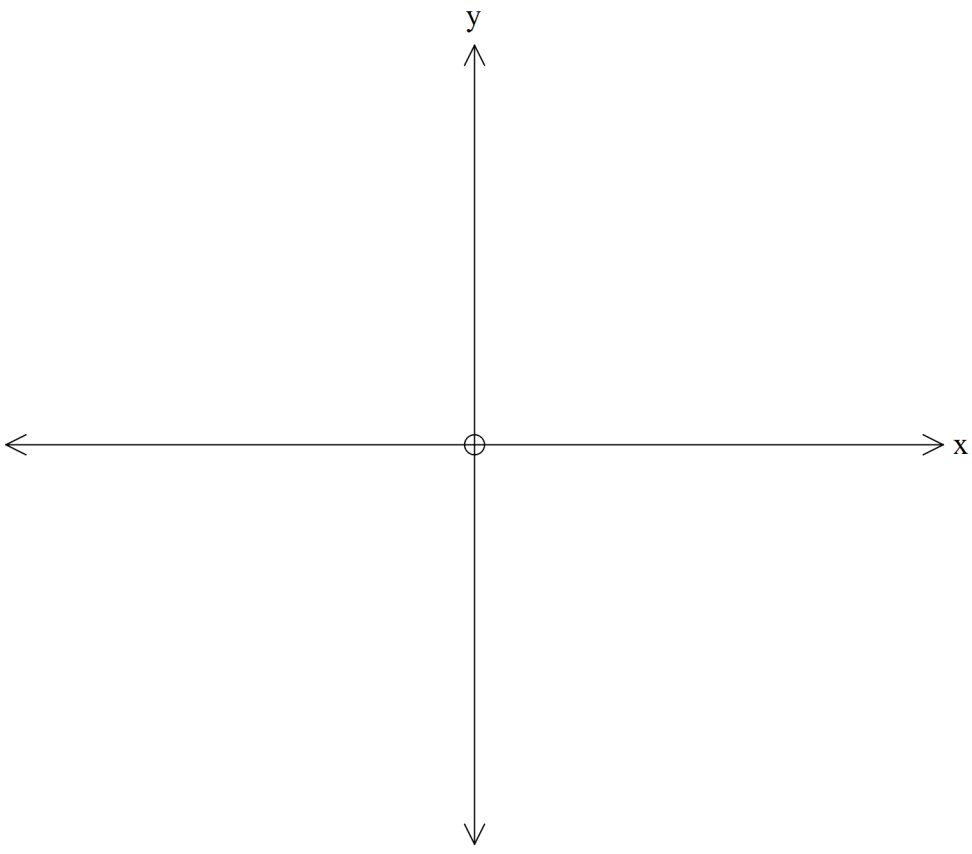
[4]

3. [6 marks]

a) Find the coordinates of all stationary points on the curve $y = (2x+1)(x-2)^4$.

[3]

b) Sketch the curve, identifying the point of inflection *(x-value sufficient)*.



[3]

4. [3 marks]

Given that $y = \sqrt[3]{x}$, use $x = 27$ and the incremental formula $\delta y \approx \frac{dy}{dx} \times \delta x$ to determine an approximate value for $\sqrt[3]{29}$.

[3]

5. [2 marks]

Given that $f'(x) = 3x^3 - 3x^2$ and $f(2) = 7$, find $f(x)$.

[2]

6. [6 marks]

Find the **antiderivative** of each of the following:

a) $2x^4$

b) $\frac{x^3}{5}$

c) $\frac{4}{x^2}$

d) e^{5x}

e) $6e^{\frac{x}{3}}$

f) $\sqrt{2x-5}$

End of Part A

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NAME: _____

Calculator Section **(26 marks)**

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7. [4 marks]

The population P of fish in a certain lake was studied over time, and at the start the number of fish was 2500.

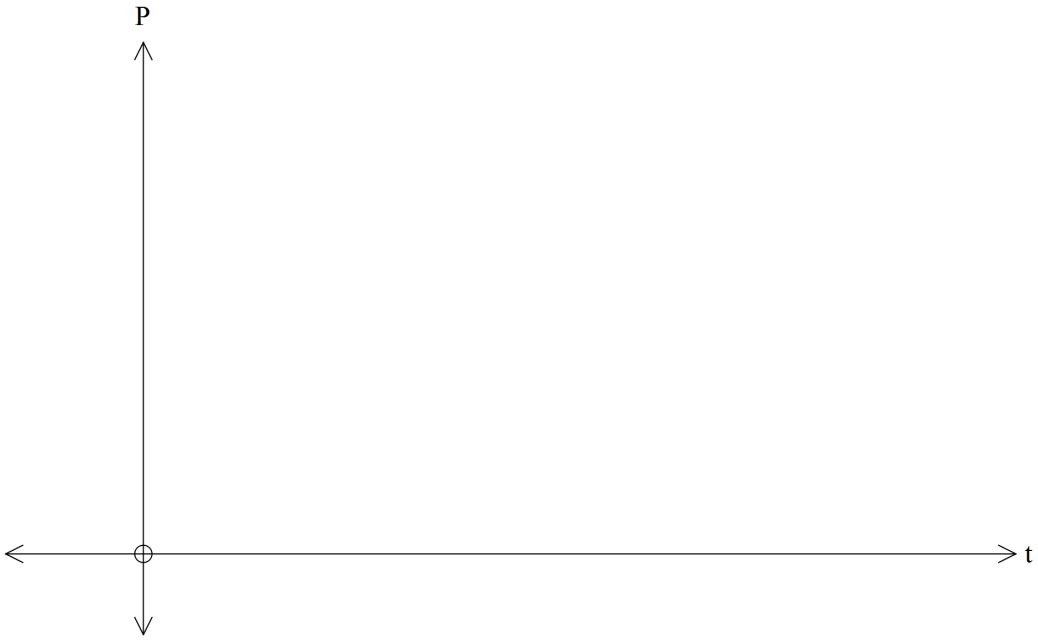
a) During the study, $\frac{dP}{dt} < 0$. What does this say about the number of fish during the study?

[1]

b) If, at the same time, $\frac{d^2P}{dt^2} > 0$, what can you say about the population rate?

[1]

c) Sketch the graph of the population P against t.



[2]

8. [2 marks]

For a certain curve, the derivative is zero when $x = -3$. Also $f''(-3) = 0$ and $f''(x) > 0$ either side of $x = -3$. Explain what kind of point is at $x = -3$.

[2]

9. [5 marks]

a) At the Blackstrap Molasses factory, the profit in dollars depends on the amount (x kg) of molasses according to the formula $P = -x^3 + 69x^2 + 5040x + 580$. What is the marginal profit after the 50^{th} kg is sold?

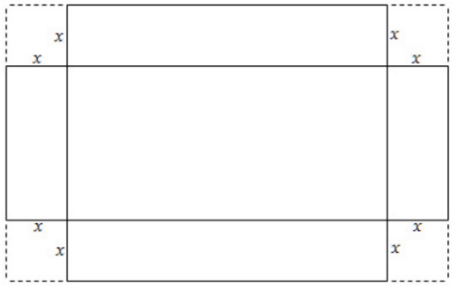
[2]

b) A spherical balloon is subjected to heat, causing it to expand uniformly. Use the incremental formula to find the approximate percentage change in volume when the diameter increases by 4%.

[3]

10. [4 marks]

A box is made by cutting square corners out of a rectangular piece of tin and folding the sides up.
If the original piece of tin measures 63 cm by 15 cm, and the squares have side length x cm, find using calculus techniques the volume of the box formed and justify that the volume is a maximum.



[4]

11. [11 marks]

A particle is initially at an origin O. It is then projected away from O and moves in a straight line such that its displacement from O, t seconds later is x metres where $x = t^3 - 6t^2 + 9t$.

Determine:

a) the initial speed of projection.

[2]

b) when the particle is at rest and how far it is from the origin at these times.

[4]

c) when the particle is moving in a positive direction.

[2]

d) the total distance travelled in the first 5 seconds.

[3]

End of Part B